Athletic Records and Human Endurance: A time-vs.-distance equation describing world-record performances may be used to compare the relative endurance capabilities of various groups of people

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Athletic Records and Human Endurance

A time-vs.-distance equation describing world-record performances may be used to compare the relative endurance capabilities of various groups of people.

Human competitive locomotive activity spans an enormous range, from a scant 50 meters sprinted in seconds to walking, running, and bicycling odysseys covering tens of thousands of kilometers and months of time. It seems to be in our nature to compete with one another, and we have developed various standard sports and distances over which to strive and be recognized for excellence.

In the course of preparing an article (1) on pace prediction for distance runners, I sought a simple way to express the relationship between distance and time of world-record runs. This subject has been investigated by many, and numerous descriptions of the time-distance curve exist (2-5). However, for the purpose of the article both accuracy and simplicity were required, and the polynomial solutions typical of most previous work were not suitable. After trying various ways of looking at the data, I noticed that, in the mile-to-marathon range (the range of interest to distance runners), a log-log plot of time vs. distance approximates a straight line (6,7). The straight line was a close fit to the data and mathematically uncomplicated, and the resulting equation was incorporated into the article.

The observation that swimmers’ loss of speed with distance roughly parallels that of runners (8) aroused my curiosity as to whether other forms of human endurance activities showed similar characteristics. I obtained world records for various forms of human locomotion (9-13) and entered them on a large-scale log-log graph of time vs. distance, as shown in Figure 1. It was apparent that in the range from about 3.5 to 230 minutes the linear relationship indeed existed for all the endurance sports investigated—running, race walking, Nordic (cross-country) skiing, roller and speed skating, cycling, freestyle swimming, and man-powered flight.

In the plot of log (time) vs. log (distance), linearity is lost at times below 3 to 4 minutes. This region of athletic competition includes sprints and other activity involving transient body processes. Hence it is not included in this analysis, which is concerned only with longer-term human endurance effort. Another region begins above 230 minutes, when the time-distance curves begin to bend upward, reflecting a slowing of the speed-time relationship that exists in the 3.5- to 230-minute region. This change may have two explanations, acting singly or jointly.

First, the body begins to tire after hours of hard work as energy stores are depleted and various other physical changes occur. Second, the region over 230 minutes is devoid of serious world-class competition. "World-class" implies here a competitor of at least Olympic finalist caliber. Beyond 24 hours, of course, the realm of multiday activity is entered. The most notable of these competitions is the Tour de France bicycle race, covering weeks of time and thousands of kilometers of hard racing. The fame and fortune accruing to the winner of this event attract the cream of the world’s professional riders, certainly placing it in the competitive range of athletics. However, because it consists of a long series of strenuous single-day efforts, no competitor can afford to treat any one day as a race in which to empty himself wholly as does, say, the Olympic marathoner.

Some reserves must be husbanded for the next day’s effort. In addition, each year sees new attempts at trans-North America runs (the current record average is over 100 km per day), long ocean swims, 24-hour running races, multiday race walks: the list seems to have no limit, as there are people in almost every sport who wish to test the true limits of their endurance. However, it is rare to see a world-class athlete compete seriously at a distance beyond the standard range of Olympic competition.

Eliminating competitive activities below 3.5 and above 230 minutes leaves a range—referred to as the "endurance range"—that includes the entire body of heavily contested human endurance events. The world records established within this range provide us with some interesting ways of looking at human stamina and how we can use the records to help measure it.

How good are the data? The best-documented sports are running, walking, and swimming, taking place as they do on accurately measured tracks and in pools. All of the other records may be viewed with some caution, but the results of world-class events are certainly not to be dismissed lightly.
The endurance equation is formulated as \( t = ax^b \), where \( t \) = time (min), \( x \) = distance (km), and \( a \) and \( b \) are constants unique for each activity. Running, men over 70 and Speed skating, men appear as a straight line, which represents time as a simple power function of distance. Examination of the values of \( b \) shows that there are great similarities within groups performing similar activities. World-class runners, men and women, have an identical fatigue factor of 1.08. Men and women swimmers share the common fatigue factor of 1.03. For running men from the ages of 40 to 70 the fatigue factor is 1.05 to 1.06. Endurance and fatigue are opposite sides of the same coin, since each indicates the absence of the other. Endurance implies an skepticism, either because they are contended on questionably measured courses, or because the contestant may elect to move along a line that may not be the shortest, in the interest of extra speed. Also, in any contest the goal is to win, and optimal race strategy may not include an all-out effort over the entire distance. I have included man-powered flight mainly for the interest it may have for the reader, since the distances covered are not precisely known, and the human performance is masked by the operational characteristics of the flying machines. However, whether or not the records used represent the absolute best, I believe that they are close to the best, and that they serve the intended analytical purpose. In any case, records are ephemeral.

The endurance equation

Within the endurance range, the logarithms of time vs. distance for each sport were best-fitted to straight lines, using least-squares technique. The curve-fitting process produced an “endurance equation,” which is a simple power function and requires knowledge of only two basic constants to describe time, speed, and distance over the endurance range for each sport (see Table 1). The equation is of the form \( t = ax^b \), where \( t \) = time, \( x \) = distance, and \( a \) and \( b \) are unique for each activity. The constant \( a \) is dependent on the units of measurement chosen and has no particular absolute significance, although it provides a measure of relative speed. The exponent \( b \), however, retains the same value regardless of the units chosen for time and distance.

The exponent \( b \) of the endurance equation may be referred to as the “fatigue factor,” because its value determines the rate at which average speed decreases with distance and time required to finish the race, thus indicating an effect of fatigue (8). Examination of the values of \( b \) shows that there are great similarities within groups performing similar activities. World-class runners, men and women, have an identical fatigue factor of 1.08. Men and women swimmers share the common fatigue factor of 1.03. For running men from the ages of 40 to 70 the fatigue factor is 1.05 to 1.06. Endurance and fatigue are opposite sides of the same coin, since each indicates the absence of the other. Endurance implies an

Table 1. The endurance equation is formulated as \( t = ax^b \), where \( t \) = time (min), \( x \) = distance (km), and \( a \) and \( b \) are constants unique for each activity

<table>
<thead>
<tr>
<th>Activity</th>
<th>( a )</th>
<th>( b )</th>
<th>Distance range (km)</th>
<th>Time range (min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Running, men</td>
<td>2.299</td>
<td>1.07732</td>
<td>1.5-42.2</td>
<td>3.5-129</td>
</tr>
<tr>
<td>Running, men over 40</td>
<td>2.569</td>
<td>1.05352</td>
<td>1.5-42.2</td>
<td>3.9-121</td>
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<tr>
<td>Running, men over 50</td>
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<td>1.05374</td>
<td>1.5-42.2</td>
<td>4.2-145</td>
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<tr>
<td>Running, men over 60</td>
<td>3.204</td>
<td>1.05603</td>
<td>1.5-42.2</td>
<td>4.9-168</td>
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<tr>
<td>Running, men over 70</td>
<td>3.854</td>
<td>1.06370</td>
<td>1.5-42.2</td>
<td>5.4-189</td>
</tr>
<tr>
<td>Running, women</td>
<td>2.598</td>
<td>1.06283</td>
<td>1.5-42.2</td>
<td>3.9-147</td>
</tr>
<tr>
<td>Swimming, men</td>
<td>9.936</td>
<td>1.02977</td>
<td>0.4-1.5</td>
<td>3.9-15</td>
</tr>
<tr>
<td>Swimming, women</td>
<td>10.578</td>
<td>1.03256</td>
<td>0.4-1.5</td>
<td>4.1-16</td>
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<tr>
<td>Nordic skiing, men</td>
<td>2.836</td>
<td>1.01421</td>
<td>15-50</td>
<td>44-149</td>
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<td>Race walking, men</td>
<td>3.565</td>
<td>1.05379</td>
<td>1.6-50</td>
<td>5.9-222</td>
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<td>Roller skating, men</td>
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<td>1.13709</td>
<td>3-10</td>
<td>5.8-22</td>
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<td>Cycling, men</td>
<td>1.015</td>
<td>1.04834</td>
<td>4-100</td>
<td>4.4-128</td>
</tr>
<tr>
<td>Speed skating, men</td>
<td>1.266</td>
<td>1.06017</td>
<td>3-10</td>
<td>4.1-15</td>
</tr>
<tr>
<td>Man-powered flight</td>
<td>3.238</td>
<td>1.10189</td>
<td>1.8-36.2</td>
<td>6.4-169</td>
</tr>
</tbody>
</table>

* Based on records up to 1 November 1979

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ability to resist fatigue, and fatigue a decline of endurance. Therefore, it seems appropriate for an endurance equation to contain a fatigue factor.

Algebraic manipulation of the endurance equation yields the following relationships for velocity vs. time and distance:

$$ v = \frac{x}{t} = \frac{t (1-b)/b}{(a/1/b)} = \frac{x(1-b)}{a} $$

Equations describing velocity were obtained directly by dividing distance by elapsed time, rather than by differentiation of the endurance equation. This procedure was used because the time-distance curve does not represent a continuous record of time elapsed as distance is covered; rather it shows, for any distance, the fastest time for covering that distance only. Speeds discussed here are all average velocities required to cover the particular distance.

In considering average speed vs. distance, we find that as distance increases, average speed falls off, at slightly differing rates for each activity (Fig. 2). Since we must assume that each contestant is doing his best, it seems likely that the natures of the various exertions are imposing different demands on the performers. In cycling, aerodynamic drag is the dominant form of resistance to the contestant, and cyclists operate in packs, taking turns breaking the wind, to help offset this. Skaters, too, operate at high air speeds, and in addition they must negotiate the many turns inherent in racing several kilometers at high speed on a 400-m oval. Runners are affected by the large forces they must develop or absorb as they overcome body inertia to move their limbs rapidly, while also raising and lowering their centers of gravity with each step. While race walkers do not take the same jarring with each step as runners do, their body motions must be more artificially contorted than those of runners, requiring great stretching effort and the use of more of their total musculature. Although swimmers do not cover the same distances as the other endurance athletes do, they are fighting a much more viscous medium, and their competitive time span is about the same as that of the skaters.

Swimming is unique as an endurance event, because it requires the competitor to abandon momentarily his rhythmic activity to make a turn every 50 m. What effect these turns have on endurance is not known. However, it is interesting to note that the continuation of the swimming time-distance curve from pool distances up to the English Channel distance (34 km) does not show much degradation. The endurance equation projects a channel crossing time of 6 hours 15 minutes for men, and 6 hours 45 minutes for women, based on world-record pool performances. The actual record at the present time is 7 hours 40 minutes, held by a woman, Penny Dean.

![Figure 2. When speed is plotted against distance for world records in various endurance sports, it becomes evident that speed decreases as distance increases in all activities. Although the cycling competition covers the greatest distance span, time of effort is less than that involved in running and race walking.](image)

While it is not contemplated that man-powered flight will soon become a hotly contested event, it does represent a new arena in which humans can contend. The 1977 flight of the Gossamer Condor, powered only by the muscles of its 60-kg pilot, former competitive cyclist Bryan Allen, won its designer, Paul MacCready, the coveted $87,000 Kremer prize for the first man-powered flight around a figure-eight-shaped, closed course (14). Two years later the descendant of this machine, the Gossamer Albatross, again piloted and powered by Allen, successfully flew the English Channel (15). Although the human performance was impressive, it is questionable whether it represents endurance at its utmost, as do more heavily contested athletic endurance events. Still, the records stand as the world's best.

The marathon run (42.2 km) is not officially recognized as possessing a "world record" by any athletic body because it is run on roads, over variable terrain, and distances are not as accurately determined as in track competitions. For example, Derek Clayton's time of 2:08:34, set in Antwerp in 1969, was recognized for 11 years as the world record by the sporting press. However, because of...
lingering questions concerning the length of the Antwerp course, the presently accepted mark is Gerard Nijboer’s 2:09:01, set in Amsterdam in 1980 (16). All other distance running records were set on the track, and most are officially recognized by the International Amateur Athletic Federation.

World-class athletes

One thing all endurance competitors have in common is that they are operating very close to the limit of steady state respiratory and circulatory performance. All breathe very hard, and all find that their ability to perform as an oxygen-conversion organism defines the limit of their excellence at their sport. They also must be masters of metering their efforts efficiently, in order to expend all their reserves just as the finish line is reached, but not before. The use of world-class athletes in the study of human endurance has the strong virtue of providing a near-absolute standard for human performance. These athletes are totally fit and motivated. It is a rare treadmill subject who could or would endure as a competitive marathoner does. Even on those occasions when a champion athlete does participate in a physiological study, his ultimate reserves are seldom explored for more than brief periods. Few researchers would push their subjects into that ill-understood realm where ability, fatigue, will, pace sense, and self-knowledge combine to produce a championship performance.

Men and women swim and run at the same distances in world-class competition. In the endurance range, the swimming events are 400, 800, and 1,500 m. The running events in Olympic competition for women end at 1,500 m; however, sufficient track and road records have been set to provide a reasonably well-established basis for comparing women to men over the entire range from 1,500 m to marathon. If the speed of a woman swimmer is compared to that of a man, it is seen that from 4 to 15 minutes the woman can attain speeds of about 94% those of a man. This time span is quite small and possibly of little use to the endurance analyst. In running, however, a picture develops over a 2-hour span, and champion women will develop about 88% of the speed of world-class men.

Women’s long-distance running is no longer in its infancy. New women’s records are frequently set, at distances from 1,500 m to marathon. Records at shorter distances have been internationally contested for decades and are no longer surpassed by a great amount when they fall. This is not true of longer distances. Norway’s Grete Waitz, holder of the current women’s marathon record (2:25:42), has herself lowered the record time by over nine minutes in her successive victories at New York in 1978, 1979, and 1980.

Age and speed

The annual publication of world running records for men over 40 (17) has made it possible to compare the performance of older men to world-class athletes in the prime of physical condition (Fig. 3). At longer distances, the speed of the fastest 40-year-old is virtually the same as that of a world-class man. Thanks to the running boom of the seventies, men at advanced ages are performing at speeds unhithought of a few decades ago: a champion septuagenarian, for example, can run 70% as fast as a world-class man. The record marathon run by a man over 70 years old was completed at a speed of 5.73 m/sec, as compared with Nijboer’s record speed of 5.45 m/sec.

Although the relative running speed of elite women vs. men is fairly constant, the relative speed of older men increases with distance (Fig. 4). This may reflect either superior endurance capability or simply a lack of speed at shorter distances. The training regimens of top-class distance runners universally incorporate “interval training,” or “repetitions,” in which they intermittently run all-out for a short distance, walk or jog to recover, and then speed up again, repeating the process many times (18). It is believed that this exhausting process leads to improved speed on long competitive runs, although actual racing speeds are lower than those employed in interval training. Few of the older men use interval training with the same degree of dedication as the world-class men do (19,20), which may explain the relatively lower performance at shorter distances, where speed is of greater importance.

In age-group competition the question of motivation must of necessity arise. Do these performances by older men really represent the best of which...
men of this age are capable? With increasing responsibilities, few older men have the time to devote to covering the hundreds of kilometers typically run each week in training by elite distance competitors. It is quite unusual to find a world-class endurance athlete who also works hard at a 9-to-5 job, as so many older runners do. It does not seem likely that an older man could possibly be trying as hard as a world-class competitor. They gain little glory, only the personal satisfaction that comes from a good performance. If substantial world recognition and acclaim were afforded to older athletes, their performance would surely become even more amazing. However, the records that currently stand represent the best that has been done, and they are certainly useful for making comparisons. They should, perhaps, be thought of as minimum indicators of what the best older men can do rather than as ultimates of performance.

What about real people? We have dealt with a composite of world records as though they represented the performances of a single person. Can any one person perform at a world-class level over the entire range of endurance? Of course. One need only remember Emil Zatopek—Olympic gold medalist at 5,000 m, 10,000 m, and marathon in Helsinki in 1952—or, more recently, the performance at Montreal of Lasse Viren, who finished fifth in the marathon after previously winning gold medals at 5,000 and 10,000 m. Eric Heiden's sweep of every available speed skating gold medal at Lake Placid in 1980 provides yet another example. It is the rule, rather than the exception, for a world-class-distance performer to excel across the entire endurance range.

Ordinary runners have also found this to be the case. In an informal survey of distance runners in Ohio, I found that each tended to perform at some constant percentage of world-class speed over the entire competitive range of distances from mile to marathon. This suggests that the runner who is performing at, say, 70% of world-class speed at 10,000 m might expect to do the same at other distances. The endurance equation for running is used with success by numerous distance runners to predict their future time at a specified distance, based on a past performance at a different distance.

Endurance is comparative rather than absolute. It is finite, its outer bound being the lifetime of the individual involved. Therefore, in considering it we must accept that at some point the individual can bear no longer under some imposed stress. Running competitions measure speed rather than endurance. A hypothetical endurance contest might involve a number of runners who follow a mechanical rabbit at a constant speed until all can no longer keep up. He who kept up the longest would be adjudged to have the greatest endurance of the competitive group, and the others would have exhibited endurance in proportion to the distance they were able to cover at the chosen speed. As in most systems involving human competition some unfairness exists. The heavier people would certainly be working harder than the lighter ones. Also, the speed chosen for the competition might favor certain of the competitors over others. Nonetheless, the contest, while admittedly inherently unfair, is not highly unfair, and it would probably be as good a way to measure competitive running endurance as any.

Specific endurance

If we accept that the endurance equation is a good description of reality, we can use it to project relative endurance between various classes of competitors. Ryder, Carr, and Herget in their discussion of the concept of "specific endurance" note that "by comparing two races run at the same or nearly the same constant speed and establishing the ratio between the distance covered in the first race and the distance covered in the second race one can arrive at a quantity we call specific endurance" (21). The elegance of the concept lies in the fact that two persons doing the same thing at the same speed may be presumed to be working at roughly the same rate, and thus specific endurance is a good measure of relative work capacity.

In order to calculate specific endurance the ranges of performance of the two classes or subjects to be compared must contain at least one common speed. When the common speed is known, the following form of the en-
A look at Figure 3 will reveal whether a comparison using specific endurance is possible between certain classes. For example, both men over 40 and men over 50 can run 5.5 m per second. The over-40 men can cover 21.9 km at this speed, while the men over 50 can cover only 3.32 km. Therefore the specific endurance of men over 50 relative to men over 40 is the ratio of the distances covered, which is 3.32/21.9, or .15.

In my opinion, the best standard for use in male endurance calculation would be the men’s world records. However, this line does not overlap all groups unless massive extrapolation is performed. Only three groups share a common speed with the world-class men—world-class women, men over 40, and men over 50 (Fig. 5). Happily, another comparison may be made which includes the entire group of men. Each group can be compared to the next younger group, as shown in Table 2. The specific endurances in the table show all too clearly the effect of age on the competitive runner: endurance beyond age 40 declines by about 20% per year. Older runners are, of course, competing mainly against others of similar age, and their finish positions, although rarely at the front of the pack, still reflect intense competitive effort with their peers.

I have avoided extrapolation beyond the limits of the endurance range because of the obvious pitfalls. However, a modest extrapolation for swimming yields a specific endurance for women swimmers relative to men of .13 to .14. This extrapolation is necessary because there is no common speed for men and women swimmers within the endurance range. The fastest woman’s speed is slower than the slowest man’s.

Use of the endurance equation gives an indication of the price that must be paid for better performance. For a world-class male runner to gain 1% in speed, he must increase his specific endurance by 13%—that is, he must become able to go 13% farther at his previous top speed at any distance. Doubling specific endurance produces only a 5% increase in speed. Perhaps this explains why a small time difference may seem to the layman to be insignificant, while to the athletes themselves the difference separates the elite from the merely good.

The endurance equation is not set forth as a fundamental theory of human locomotion. It is only a simple description of what is observed. I have tried to avoid speculation regarding physiological reasons for the effects noted in this article, preferring only to describe the reality as I have seen it. There are numerous ways in which equations may be fitted to data, and my choice of a power function is based on the desire to have an expression that represents a good compromise between accuracy and simplicity. I believe that the expression does represent such a compromise, as it is easily manipulated yet still projects times that come close to the world records at any distance within the endurance range. I hope that it will be a useful tool for those concerned with the application of the time-distance relationship to their own work or interests.

References

Table 2. Common speeds and specific endurances among older men

<table>
<thead>
<tr>
<th></th>
<th>Range of common speed (m/ sec)</th>
<th>Range of specific endurance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Men over 40 vs. world-class men</td>
<td>6.35–5.43</td>
<td>.270–.663</td>
</tr>
<tr>
<td>Men over 50 vs. men over 40</td>
<td>5.74–5.31</td>
<td>.152–.151</td>
</tr>
<tr>
<td>Men over 60 vs. men over 50</td>
<td>5.09–4.80</td>
<td>.105–.100</td>
</tr>
<tr>
<td>Men over 70 vs. men over 60</td>
<td>4.45–4.22</td>
<td>.091–.081</td>
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x = (av)1/(1–b)

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