Problem 1 (15 pts). Determine whether the following series are absolutely convergent, conditionally convergent, or divergent.

1. \[ \sum_{n=0}^{\infty} \frac{(-3)^n}{n!} \]

2. \[ \sum_{n=1}^{\infty} \frac{n^n}{3^n+3n} \]

3. \[ \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{5 \cdot 8 \cdot 11 \cdots (3n+2)} \]

Problem 2 (10 pts). Fix the following statements/definitions:

1. A series \( \sum_{n=0}^{\infty} a_n \) is absolutely convergent if and only if \( \lim_{n \to \infty} a_n = 0 \).

2. In the root test, we check \( \lim_{n \to 0} |a_n|^{1/n} \): if it’s 0, then the series is absolutely convergent.

3. A series which is conditionally convergent has the property that rearranging terms in the summation automatically changes the sum of the series.

4. If a series is absolutely convergent, then it is certainly conditionally convergent.

Problem 3 (10 pts). For the following two power series, calculate the intervals and radii of convergence.

1. \[ \sum_{n=1}^{\infty} \frac{(x - 2)^n}{3^n} \]

2. \[ \sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n - 1)!} \]

Problem 4 (15 pts). Given \( f(x) = \ln(x) \). Use your TI as well as possible to do the following:

1. Approximate \( f \) by a Taylor polynomial with degree 3 centered at \( a = 4 \).

2. Use Taylor’s inequality to estimate the accuracy of the approximation \( f(x) \approx T_3(x) \) when \( x \) lies in the interval \( 3 \leq x \leq 5 \).

3. Check your result in part (2) by graphing \( |R_3(x)| \).
Problem 5 (10 pts). Using the power series for the function

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \quad (|x| < 1)$$

compute power series for the following functions. Show your work! Of course you could just use your TI, but don’t! I want you to make use of the series above. Include the intervals of convergence.

1. \(f(x) = \ln(1 + 2x)\)
2. \(f(x) = (1 + 2x)^{-3}\)

Recap of Calc II

You should do eight (8) of the following ten (10) five-point problems. This means that you may skip two (2) of them. In large letters write “SKIP!” over the two you wish to skip.

Problem 6 (5 pts). Compute the integral \(\int_{0}^{\pi} x \ln(x) dx\) by parts, showing all details.

Problem 8 (5 pts). Approximate (by hand - i.e., write out the coefficients, etc.) the integral

$$\int_{-1}^{1} \sqrt{1 + x^3} \, dx$$

using Simpson’s rule with \(n = 8\).

Problem 14 (5 pts). Find a good bound on the error incurred by approximating the series

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{(1 + n)^2}$$

by the partial sum including all terms up to the term \(a_m\).

Problem 15 (5 pts). Use the integral comparison test to demonstrate that the series

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

diverges.