MAT212 Test 3 (Spring 2004)  
Sections 12.1,2,4 and 13.1,2,4

Name:

Directions:

- ALL ANSWERS to be graded MUST BE ON THIS TEST.
- Show ALL WORK to receive ANY CREDIT.
- Table values (normal and t) are attached to your test. Explain what you use, when you use it!
- Points for each problem are in parentheses.

Good luck!

Problem 1 (25 points) The population of a small town (10,000 citizens) has been devastated by the flu. A random sample of 100 citizens finds that 42% have been afflicted.

1. (15 points) Estimate with 95% confidence the true proportion of all citizens who have had the flu. Include an interpretation of your interval.

2. (4 points) Verify that the above procedure is valid (i.e., are the conditions for this procedure met?).

3. (6 points) How many citizens would need to be sampled to estimate the proportion afflicted to within a margin of error of 2% when using a 95% level of confidence?
Problem 2: (24 points) A government official is charged with ensuring that the fall election goes well. She is concerned that there be enough staff at the election booths to handle the crowds. She conjectures that turn-out in the fall election will be 40%. A random survey of 150 registered voters designed to test her hypothesis finds that 63 of them plan on voting.

1. (4 points) Identify the null and alternative hypothesis for this problem.

2. (5 points) Identify and compute the appropriate statistic to use for the test.

3. (5 points) Compute the appropriate rejection region, and draw the appropriate conclusion for the test if the level of significance is .05.

4. (5 points) Give an interpretation based on your results.

5. (5 points) Describe a Type II error in terms of this problem.
Problem 3 (15 points) At your company employee sales objectives for each quarter are established. You wonder if they are unrealistically high, and so obtain a sample of 100 employees’ objectives and their actual performance. We compute the difference of the two (objective minus actual sales) for each employee, so that we’re left with 100 differences (measured in thousands of dollars).

- (5 points) What might your conclusion be if the mean difference were negative?

- (10 points) Suppose that the following output describes the differences:

  Descriptive Statistics: diff
  Variable     N   Mean   StDev
  diff      100  19.138  9.622

  Do the statistics support your hypothesis? Do that test!
Problem 4 (16 points) A marketing survey finds that two different clients are spending what appears to be quite different amounts of money on advertising campaigns:

2. Company 2: $43500 average, sample standard deviation of 12800, 43 samples.

You wonder if the difference in costs is statistically significant.

1. (5 points) What is the appropriate statistic to use, and what is its value?

2. (5 points) What is your decision for the hypothesis test?

3. (6 points) What is your interpretation of the results?
**Problem 5** (20 points) We are interested in knowing if there are male/female differences in salary. In order to test this, 100 pairs of adult fraternal twins (male/female pairs) are surveyed with the following results:

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
</tr>
</thead>
<tbody>
<tr>
<td>males</td>
<td>100</td>
<td>42074</td>
<td>12276</td>
</tr>
<tr>
<td>females</td>
<td>100</td>
<td>38855</td>
<td>10854</td>
</tr>
<tr>
<td>Difference</td>
<td>100</td>
<td>5219</td>
<td>17235</td>
</tr>
</tbody>
</table>

1. (4 points) State the null and alternative hypotheses for this problem.

2. (4 points) What is the test statistic?

3. (4 points) What is the p-value for this test?

4. (4 points) Based on your p-value, give your decision for the test.

5. (4 points) Why were twins used?
Table 1: T-table giving probability $P(T > t)$ (normal values are given under the $\infty$ degrees of freedom).

$$
\begin{array}{l|llllllllllll}
\text{df} & 1.1 & 1.2 & 1.3 & 1.4 & 1.5 & 1.6 & 1.7 & 1.8 & 1.9 & 2.0 & 2.1 & 2.2 & 2.3 \\
\hline
68 & 0.138 & 0.117 & 0.099 & 0.083 & 0.069 & 0.057 & 0.047 & 0.038 & 0.031 & 0.025 & 0.020 & 0.016 & 0.012 \\
69 & 0.138 & 0.117 & 0.099 & 0.083 & 0.069 & 0.057 & 0.047 & 0.038 & 0.031 & 0.025 & 0.020 & 0.016 & 0.012 \\
98 & 0.137 & 0.117 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.012 \\
99 & 0.137 & 0.117 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.012 \\
100 & 0.137 & 0.116 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.012 \\
103 & 0.137 & 0.116 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.012 \\
104 & 0.137 & 0.116 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.012 \\
105 & 0.137 & 0.116 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.012 \\
149 & 0.137 & 0.116 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.011 \\
150 & 0.137 & 0.116 & 0.098 & 0.082 & 0.068 & 0.056 & 0.046 & 0.037 & 0.030 & 0.024 & 0.019 & 0.015 & 0.011 \\
\infty & 0.136 & 0.115 & 0.097 & 0.081 & 0.067 & 0.055 & 0.045 & 0.036 & 0.029 & 0.023 & 0.018 & 0.014 & 0.011 \\
\end{array}
$$

Table 2: Critical values of the normal.

$$
t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}}
$$

$$
s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}
$$

$$
\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2, df} \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}
$$

$$
t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{s_2^2}{n_2} \right)}}
$$

$$
\bar{x}_1 - \bar{x}_2 \pm z_{\alpha/2} \sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{s_2^2}{n_2} \right)}
$$

$$
df = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\left( \frac{s_1^2/n_1}{n_1-1} + \frac{s_2^2/n_2}{n_2-1} \right)}
$$

$$
z = \frac{\hat{p} - p}{\sqrt{p(1-p)/n}}
$$

$$
\hat{p} \pm z_{\alpha/2} \sqrt{\hat{p}(1-\hat{p})/n}
$$

$$
n = \left( \frac{z_{\alpha/2} \sqrt{p(1-p)}}{W} \right)^2
$$

$$
t = \frac{\bar{x} - \mu}{s/\sqrt{n}}
$$