MAT 212 – Solutions to several hypothesis testing problems.

11.45 1) \( H_0 : \mu = 5000 \quad H_1 : \mu > 5000 \)

\[ z = \frac{\bar{x} - 5000}{400 / \sqrt{n}} \]

2) \( z = \frac{5065 - 5000}{400 / \sqrt{100}} = 1.62 \)

3) \( p\text{-value} = P(Z > 1.62) = .5 - .4474 = .0526 \)

4) Since 0.0526 > 0.05, we fail to reject \( H_0 \).

5) There is weak evidence to conclude that the mean lifetime of these bulbs exceeds 5000 hours.

11.47 1) \( H_0 : \mu = 560 \quad H_1 : \mu > 560 \)

\[ z = \frac{\bar{x} - 560}{50 / \sqrt{n}} \]

2) \( z = \frac{569 - 560}{50 / \sqrt{20}} = .81 \)

3) \( p\text{-value} = P(Z > .81) = .5 - .2910 = .2090 \)

4) Since 0.2090 > 0.05, we fail to reject \( H_0 \).

5) There is no evidence to conclude that the mean GMAT score of MBA applicants is more than 560.

11.52 1) \( H_0 : \mu = 20 \quad H_1 : \mu < 20 \)

\[ z = \frac{\bar{x} - 20}{3 / \sqrt{n}} \]

2) \( z = \frac{19.39 - 20}{3 / \sqrt{36}} = -1.22 \)

3) \( p\text{-value} = P(Z < -1.22) = .5 - .3888 = .1112 \)

4) Since 0.1112 is not smaller than 0.05, we fail to reject \( H_0 \).

5) There is no evidence to infer that the number of customers will decrease.

11.53 1) \( H_0 : \mu = 100 \quad H_1 : \mu > 100 \)

\[ z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} \]

2) \( z = \frac{105.7 - 100}{12 / \sqrt{100}} = 4.75 \)

3) \( p\text{-value} = 0.5 - 0.4990 = 0.0010 \)

4) Decision: Reject the null hypothesis.

5) There is strong evidence to infer that the site is acceptable (more than 100 pedestrians pass by the location per hour).

11.56 1) \( H_0 : \mu = 32 \quad H_1 : \mu < 32 \)

\[ z = \frac{\bar{x} - 32}{6 / \sqrt{n}} \]

2) \( z = \frac{29.92 - 32}{6 / \sqrt{110}} = -3.64 \)

3) \( p\text{-value} = P(Z < -3.64) < 0.0010 \)

4) (5) Reject the null hypothesis.

5) There is enough evidence to infer that the mean time away from desks is less than 32 minutes.

A type I error occurs when we conclude that the plan decreases the mean time away from desks when it actually does not. This error is quite expensive. Consequently we demand a low p-value (we set \( \alpha \) low, which forces the p-value to be small in order to reject \( H_0 \)). The p-value is small enough to infer that there has been a decrease.