X is the lifetime for a randomly selected bulb where \( X \sim \text{Normal}(\mu = 5100, \sigma = 200) \).

\[
\begin{align*}
P(X > 5000) &= P\left(\frac{X - 5100}{200} > \frac{5000 - 5100}{200}\right) = P(Z > -0.5) \\
&= P(-0.5 < Z < 0) + P(Z > 0) = 0.1915 + 0.5 = 0.6915
\end{align*}
\]

Working backwards, we want to find \( x \) such that \( P(X > x) = 0.9800 \).

So in the standard normal picture, what value of \( z \) gives \( P(Z > z) = 0.98 \)? Look for 0.4800 in the body of Table 3. Use the closest value which is 0.4798. This corresponds to the little \( z \) value of 2.05, but recall in our picture, we are below 0 (negative numbers), so we add a negative sign. Thus, \( P(Z > -2.05) = 0.98 \). Notice we are looking for \( z_{0.98} = -z_{0.02} \).

Now, we “untransform” our -2.05 to find the value of \( x \) we are seeking.

Since \( z = \frac{x - 5100}{200} \) we can put in -2.05 for \( z \) and solve for \( x \).

\[
-2.05 = \frac{x - 5100}{200}
\]

which implies that \( 200(-2.05) + 5100 = x \) and hence \( x = 4690 \).

We would advertise that the bulbs last \( 4,690 \text{ hours} \).
8.42 a \( P(X > 12000) = P\left( \frac{X - \mu}{\sigma} > \frac{12000 - 10000}{2400} \right) = P(Z > .83) = .5 - P(0 < Z < .83) = .5 - .2967 = .2033 \)

\[ \text{Standard Normal Distribution} \]

\[
\begin{array}{c}
\text{f(x)} \\
0.0 \\
0.1 \\
0.2 \\
0.3 \\
0.4 \\
\end{array}
\]

\[ -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

b \( P(X < 9000) = P\left( \frac{X - \mu}{\sigma} < \frac{9000 - 10000}{2400} \right) = P(Z < -.42) = .5 - P(0 < Z < .42) = .5 - .1628 = .3372 \)

\[ \text{Standard Normal Distribution} \]

\[
\begin{array}{c}
\text{f(x)} \\
0.0 \\
0.1 \\
0.2 \\
0.3 \\
0.4 \\
\end{array}
\]

\[ -4 \quad -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \]
8.43 \( P(0 < Z < z_{.001}) = .5 - .001 = .4990; \ z_{.001} = 3.08 \). Then, \( z_{.001} = \frac{x - \mu}{\sigma}; \ 3.08 = \frac{x - 10000}{2400} \); \( x = 17392 \).

\[ \text{Standard Normal Distribution} \]

8.44 a \( P(X > 70) = P\left(\frac{X - \mu}{\sigma} > \frac{70 - 65}{4}\right) = P(Z > 1.25) = .5 - P(0 < Z < 1.25) \)

\[ = .5 - .3944 = .1056 \]

b \( P(X < 60) = P\left(\frac{X - \mu}{\sigma} < \frac{60 - 65}{4}\right) = P(Z < -1.25) = .5 - P(0 < Z < 1.25) \)

\[ = .5 - .3944 = .1056 \]

c \( P(55 < X < 70) = P\left(\frac{55 - 65}{4} < \frac{X - \mu}{\sigma} < \frac{70 - 65}{4}\right) = P(-2.50 < Z < 1.25) \)

\[ = P(0 < Z < 2.50) + P(0 < Z < 1.25) = .4938 + .3944 = .8882 \]

8.45 a \( P(X < 70000) = P\left(\frac{X - \mu}{\sigma} < \frac{70000 - 82000}{6400}\right) = P(Z < -1.88) = .5 - P(0 < Z < 1.88) \)

\[ = .5 - .4699 = .0301 \]

b \( P(X > 100000) = P\left(\frac{X - \mu}{\sigma} > \frac{100000 - 82000}{6400}\right) = P(Z > 2.81) = .5 - P(0 < Z < 2.81) \)

\[ = .5 - .4975 = .0025 \]

8.48 \( P(X > 8) = P\left(\frac{X - \mu}{\sigma} > \frac{8 - 7.2}{.667}\right) = P(Z > 1.2) = .5 - P(0 < Z < 1.2) = .5 - .3849 = .1151 \)

8.49 \( P(0 < Z < z_{.25}) = .5 - .25 = .2500; \ z_{.25} = .67; \)

\[ z_{.25} = \frac{x - \mu}{\sigma}; \ .67 = \frac{x - 7.2}{.67}; \ x = 7.65 \text{ hours} \]
8.54  
\( a \ P(X > 30) = P\left( \frac{X - \mu}{\sigma} > \frac{30 - 27}{7} \right) = P(Z > .43) = .5 - P(0 < Z < .43) \)
\[ = .5 - .1664 = .3336 \]
\( b \ P(X > 40) = P\left( \frac{X - \mu}{\sigma} > \frac{40 - 27}{7} \right) = P(Z > 1.86) = .5 - P(0 < Z < 1.86) \)
\[ = .5 - .4686 = .0314 \]
\( c \ P(X < 15) = P\left( \frac{X - \mu}{\sigma} < \frac{15 - 27}{7} \right) = P(Z < -1.71) = .5 - P(0 < Z < 1.71) \)
\[ = .5 - .4564 = .0436 \]
\( d \ P(0 < Z < z_{.20}) = .5 - .20 = .3000; \ z_{.20} = .84 \)
\[ z_{.20} = \frac{x - \mu}{\sigma}; \ .84 = \frac{x - 27}{7}; \ x = 32.88 \]

8.56  
\( a \ P(X < 10) = P\left( \frac{X - \mu}{\sigma} < \frac{10 - 16.40}{2.75} \right) = P(Z < -2.33) = .5 - P(0 < Z < 2.33) \)
\[ = .5 - .4901 = .0099 \]
\( b \ P(-z_{.10} < Z < 0) = .5 - .10 = .4000; \ -z_{.10} = -1.28 \)
\[ -z_{.10} = \frac{x - \mu}{\sigma}; \ -1.28 = \frac{x - 16.40}{2.75}; \ x = 12.88 \]